Shear migration and chaotic mixing of particle suspensions in a time-periodic lid-driven cavity

B. Xu and J. F. Gilchrist

Department of Chemical Engineering, Lehigh University, Bethlehem, Pennsylvania 18015, USA

(Received 28 July 2009; accepted 15 March 2010; published online 5 May 2010)

This work simulates segregation of noncolloidal particle suspensions in a two dimensional time-periodic flow. Two different mixing protocols having alternating moving boundaries in a cavity known to generate chaotic advection while maintaining a constant energy input rate are applied to each suspension. A diffusive flux model is used to capture the essence of shear-induced migration. In this system, fluid flow drives both mixing and segregation where the local rheology is a function of particle volume fraction. The impact of migration strength, altered by varying the particle size and bulk volume fraction, and topology, altered by breaking symmetry in the flow when varying the period length, are investigated. As a result of the complex interplay between the flow topology and shear migration, the concentration profile ranges from that representing the underlying topology to that of steady flow in a lid-driven cavity and depends on the parameters mentioned above and the structure produced by the two mixing protocols. In this system, increasing the size of chaotic regions does not result in enhancing mixing. These results challenge conventional wisdom in designing small scale flows for mixing and separations in microscale applications. © 2010 American Institute of Physics. [doi:10.1063/1.3394981]

I. INTRODUCTION

Shear-induced migration is a distinct feature of flowing concentrated particle suspensions. While two body interactions of perfect spheres are reversible under Stokes flow, irreversible migration of particles resulting in demixing arises at moderate particle concentrations. Despite extensive research in this area, shear-induced migration has primarily been investigated in classical simple flows, e.g., pressure-driven flow between parallel plates and in conduits and Couette flow. The general heuristic is particles migrate from high shear rate regions to low shear rate regions. Interplay between this migration, causing gradients in volume fraction \( \phi \), and the concentration-dependent suspension rheology distorts the flow from that of Newtonian fluids. There is also notable work on nonsteady simple oscillatory shear and pressure-driven flows. In a select few studies, shear migration in more complicated geometries that have been explored include steady flows in rectangular lid-driven cavities, symmetric and asymmetric channel bifurcations, and open cavity flows. Recent investigations have expanded to include complex flows exhibiting instability due to shear migration and chaotic advection, utilizing the fact that shear migration of a given suspension is stronger in microfluidic applications. However, how underlying flow topology (e.g., Ref. 17) mediates or enhances shear migration and how shear migration influences the flow topology is unclear. Developing this understanding is critical since the vast majority of industrial and natural suspension flows exhibit transient and chaotic properties.

In reality more complex time dependency may be encountered, including that which can induce chaotic advecting. Chaos is widely used to enhance mixing, especially under low-Reynolds-number conditions, when turbulence is inhibited. In this work, we refer to “chaos” as continuum transport property as opposed to the chaotic motion of individual constituents leading to diffusion and migration. It can exist when symmetry is broken in a three dimensional (3D) flow or when two dimensional (2D) or 3D flows undergo time modulation. Chaotic advection in fluid flow is complex, and many physical systems have increasing degrees of complexity including scalar (heat or mass) transport and chemical reactions. Interplay between chaotic mixing and segregation driven by body forces (e.g., Refs. 18 and 19) is more complex, and the interplay between chaos and rheology-driven migration is perhaps the most complex. Permutations of combinations of these effects become increasingly complex (e.g., heat transport and reaction in chaotic flows). Studies considered the effects of shear thinning and elasticity on the flow topology of chaotic advection without migration or phase change. The general assumption is chaotic advection, without the increase in shear rate or energy input, would mediate shear migration-driven segregation and enhance dispersion. In contrast, granular flows are known to segregate readily due to body force-driven demixing during chaotic advection in 2D and 3D (Ref. 24) flows.

We study the behavior of suspensions with varying degrees of shear migration in a prototypical chaotic flow known as time-periodic lid-driven cavity flow. This allows for the following two independent modes of probing the interplay between chaotic advection and segregation resulting from shear migration. First, we can adjust the topology of the flow without increasing the energy input into the system. This breaks the symmetry of the flow and tunes the time periodicity to generate very weakly chaotic to chaotic

---

Electronic mail: gilchrist@lehigh.edu.
advection-dominated flows. Second, we can adjust the suspension properties including relative particle size and average volume fraction to tune the rate of migration. In complex small scale flows often found in microelectromechanical systems (MEMS) when the ratio of particle to channel diameter is larger, the effects of migration are magnified. This study aims to generate a set of heuristics by which one can better understand the effect of this interplay between rate of migration and the rate of chaotic mixing in the generation of nontrivial segregation patterns.

II. FORMULATION

Widely used models that have been proposed fall into two categories: the diffusive flux approach and the suspension stress approach. Each of these models has limitations, including neglecting particle-wall interactions and excluded volume and singularity at localized zero shear rate and high curvature. For simplicity of implementation, noncolloidal spherical particles dispersed in a Newtonian fluid are modeled by a diffusive-flux approach, where the change in local volume fraction $\phi$ is governed by

$$\frac{D\phi}{Dt} = a^2 K_c \nabla \cdot (\phi^2 \nabla \gamma + \phi \gamma \nabla \phi) + a^2 K_s \nabla \cdot \left( \phi \gamma \phi^2 \frac{1}{\eta} \frac{\partial \eta}{\partial \phi} \nabla \phi \right),$$

(1)

where $a$ is particle radius, $\gamma$ is shear rate, and $K_c = 0.41$ and $K_s = 0.62$ are constants. Note that for a characteristic system size of $L$, the characteristic rate of migration is proportional to $\lambda = (a/L)^2$. Therefore, migration in a given suspension is enhanced in small-scale flows often found in MEMS applications. The viscosity of the suspension is approximated by the Krieger–Dougherty relation

$$\eta = \eta_0 \left(1 - \frac{\phi}{\phi_m}\right)^{-1.82},$$

(2)

where $\eta_0$ is the viscosity for the interstitial Newtonian fluid.

Neumann boundary condition is applied to ensure no flux across any wall and $\phi$ is assumed a uniform value $\phi_{ave}$ at $t=0$. In relation to the limitations described above, this model does not consider curvature-driven flux resulting from particle interactions on neighboring curved trajectories. In select simulations incorporating this flux, the effect of curvature-induced migration is strongest near the corners strengthening the near-corner concentration gradients presented here. The continuum description of the behavior of discrete particles for increasing $\lambda$ is challenged in these regions, not considered in any of the above models.

It is shown that within the Stokes-flow regime, 2D steady flow results only in simple closed streamlines. Chaotic advection can be achieved by applying periodic driving forces to the fluid. Examples of closed flow exhibiting chaos include journal-bearing flow, lid-driven rectangular-cavity flow, and egg-beater flow. In this work, we consider the 2D time-periodic lid-driven cavity flow in a rectangular cavity with height $L$ and an aspect ratio 2:1. As is shown in Fig. 1, the top and bottom walls are sliding alternately with equal velocity, for some equal length of time. We normalize time length by dividing by a characteristic time $t_{slide} = L/v_{slide}$. Therefore, $t_{slide} = 1$ is the time for an element on the wall to traverse $L$. Another parameter that affects mixing is the moving wall direction. Two different situations are possible: when the top and bottom walls slide in opposite directions (e.g., top wall from left to right, then bottom wall from right to left), it is called $S_1$; when the two walls move in the same direction (e.g., top wall from left to right, then bottom wall), we call it $S_2$. The length for a full cycle of alternating flow is defined as a “period” $2T$. Assuming a vanishingly small Reynolds number, the momentum transport of the flow is given by

$$-\nabla p + \nabla \cdot [\eta(\nabla \mathbf{v} + \nabla \mathbf{v}^T)] = 0$$

(3)

and

$$\nabla \cdot \mathbf{v} = 0.$$  

(4)

The boundary condition can be described as

$$v_{top} = v_{bottom} = 0, \quad 2nT < t < (2n+1)T;$$

(5)

$$v_{top} = 0, v_{bottom} = \pm v_{slide}, \quad (2n+1)T < t < 2(n+1)T,$$

(6)

where on the second line, the plus sign stands for $S_2$ and minus sign for $S_1$ and $n = 0, 1, 2, \cdots$.

We simulate the evolution of concentration and velocity in time (COMSOL MULTIPHYSICS 3.4). The flow velocity and pressure are represented by 2D $P^2P^1$ (Crouzeix-Raviart) elements and concentration by 2D quadratic Lagrange elements. Equations (1)–(4) combined with the boundary conditions are solved by the GMRE solver. We varied different values for particle radius $\lambda$, period $T$, and average volume fraction $\phi_{ave}$ to investigate their impact on particle distribution under the two different scenarios, $S_1$ and $S_2$, previously defined.
Sufficient to drive significant concentration gradients. 15,16 Studies of shear migration investigate increasing degrees of migration transients associated with the start or stop of the wall at corners to just below the center of the cavity. There are no bulk volume fraction gradients resulting from migration, the horizontal symmetry for the underlying flow and this material convection forms the central high concentration island. This segregation resulting from shear migration has significant ramifications regarding heat or mass transport from the walls to the center.

III. RESULTS

Steady lid-driven cavity flow is studied at an average bulk volume fraction \( \phi_{\text{ave}} = 0.2 \). Although many previous studies of shear migration investigate \( \phi_{\text{ave}} > 0.4, \phi_{\text{ave}} > 0.1 \) is sufficient to drive significant concentration gradients. 15,16 Figure 2(a) shows the steady-state shear rate profile from the lid-driven cavity flow at \( \lambda = 0.01 \). Despite the strong viscosity gradients resulting from migration, the horizontal symmetry suggests only a small deviation (6%) of \( \nabla \cdot \mathbf{v} \) from a Newtonian fluid. As expected, \( \dot{\gamma} \) is highest near the upper moving boundary and lowest in the lower corners opposite the moving boundary. A band of lower \( \dot{\gamma} \) extends from the upper corners to just below the center of the cavity. There are no transients associated with the start or stop of the wall at \( \text{Re} = 0 \).

Concentration profiles for steady cavity flows with increasing degrees of migration \( \lambda = 1 \times 10^{-4}, 2.5 \times 10^{-3}, \) and \( 1 \times 10^{-2} \) are shown in Figs. 2(b)–2(d) to demonstrate the effect of migration in this shear profile in the absence of chaotic advection. The concentration scale bar in these plots is the same \([0,0.4]\) for direct comparison and shows that these results are nearly identical to previous simulations. 10 For relatively weak migration, \( \lambda = 1 \times 10^{-4} \), the steady concentration profile is uniform with the exception of two bands of slightly higher and lower concentration extending from the upper corners to the lower boundary. Concentration gradients are slightly larger on the right side of the cavity downstream of the flow near the upper wall. The closed streamlines of this flow inhibit migration toward the center. With increasing degree of migration the concentration gradients become more pronounced and more asymmetric with respect to a reflection across the vertical midplane. At \( \lambda = 2.5 \times 10^{-3} \) particles migrate further into the center and more strongly into the lower corners. The local minimum in shear rate, which forms a curved valley extending from center to the two upper corners [Fig. 2(a)], results in a local maximum concentration region as well. Instead of sitting exactly in the low shear rate basin, however, this high-concentration band is deformed and transported by convection forming discernible asymmetry. At \( \lambda = 1 \times 10^{-2} \) the island in the center exists as well, but it has more pronounced asymmetry with particles concentrated on the left side and a lobe of low concentration near the right wall. Conditions approach the same with \( \phi_{\text{ave}} = 0 \) and \( \phi_{\text{ave}} = 0.4 \) in localized low and high concentration regions of the flow. The bottom corners have two spires having the highest concentration that point toward the upper corners. The same high concentration regions are also found in the previous study. 10 Generally, the particles migrate from regions of high shear rate to regions of low shear rate; this migration breaks the symmetry that would exist without migration in the underlying flow and this material convection forms the central high concentration island. This segregation resulting from shear migration has significant ramifications regarding heat or mass transport from the walls to the center.

We now consider the effect of migration in chaotic flows. Figures 3 and 4 show the simulations results of 16 different flows for \( S_1 \) and \( S_2 \), respectively. \( T \) increases from top to bottom, effectively increases the degree of chaotic mixing. Increasing \( T \) also means that the system spends more time as a steady flow during each half-cycle of the period. From left to right the degree of migration increases. The first column is the Poincaré or stroboscopic map of the Newtonian flow with increasing \( T \). This is produced by advecting a number of initial positions and selectively plotting their locations at the end of each cycle. This results in a cross section of the underlying dynamics and highlights invariant features of the flow including elliptical periodic points at the center of regular regions, unstable periodic points that can result in regions of chaotic mixing, and closed trajectories and chains of periodic orbits that form KAM surfaces 15,36 which both act as barriers to transport. The columns to the right of the Poincaré maps are concentration profiles with increasing strength of migration and decreasing \( \lambda \). Each column has a scale bar highlighting concentration gradients and allowing comparison with the underlying flow portrayed by the corresponding Poincaré map.

For \( S_1 \), the boundary conditions of the two half periods of the flow have rotational symmetry through the center. For Newtonian fluids the topology of the flow has \( x \)-axis symme-
try as seen in the Poincaré maps. For $T=3/4$ [Fig. 3(a)] a regular region exists to the right of center and most of the flow is non- or weakly chaotic. With migration at $\lambda=1 \times 10^{-4}$ [Fig. 3(b)], the Poincaré map’s underlying structure is apparent and the concentration profile has similar features to those resulting from migration in steady flow. The lowest concentration is at the boundaries, and higher concentration rings are found just inside the boundary and slightly right of the center of the channel. The invariant structures surrounding the center island are barriers to transport. At $\lambda=2.5 \times 10^{-3}$ [Fig. 3(c)] the concentration profile resulting from stronger shear migration shows features deviating from the underlying topology. The central ring is much higher in concentration and lines of higher concentration extend from each corner toward the center. At $\lambda=1 \times 10^{-2}$ [Fig. 3(d)] shear migration pushes most particles in suspension toward

![Poincaré map](image)

**FIG. 3.** (Color online) Poincaré maps and concentration profiles of suspensions in $S_1$. The vertical direction probes the influence of time $T$ the half period of the cycle, while the horizontal direction examines increasing degree of migration. The left column is Poincaré maps representing the evolution of initial conditions plotted after every period $2T$, with vertically symmetric topology stemming from the symmetry of the boundary conditions. The second, third, and fourth columns are concentration profiles plotted at the end of each cycle demonstrating the topology of segregation with $\lambda=1 \times 10^{-4}$, $2.5 \times 10^{-3}$, and $1 \times 10^{-2}$. The topology represented in the Poincaré map is visible at low $T$ and $\lambda$. High $T$ and $\lambda$ result in segregation profiles that mimic the steady profile seen in Fig. 2(d) rotated 180°.

![Poincaré map](image)

**FIG. 4.** (Color online) Poincaré maps and concentration profiles of suspensions in $S_2$. The vertical direction probes the influence of time $T$ the half period of the cycle, while the horizontal direction examines increasing degree of migration. The left column is Poincaré maps representing the evolution of initial conditions plotted after every period $2T$, with 180° rotationally symmetric topology stemming from the symmetry of the boundary conditions. The second, third, and fourth columns are concentration profiles plotted at the end of each cycle demonstrating the topology of segregation with increasing rates of shear migration $\lambda=1 \times 10^{-4}$, $2.5 \times 10^{-3}$, and $1 \times 10^{-2}$. The topology represented in the Poincaré map is visible at low $T$ and $\lambda$. High $T$ and $\lambda$ result in segregation profiles that when vertically reflected mimic the steady profile seen in Fig. 2(d) and when horizontally reflected mimic Fig. 3(p).
the middle with the exception of thin lines of higher concentration near the upper left and lower corners oriented toward corresponding higher-concentration lines extending from the center.

For $S_1$ and $T=3/2$ [Figs. 3(e)–3(h)] the Poincaré map looks significantly different from $T=3/4$. A regular region exists right of center; however, here the flow topology is dominated by a period-5 chain of islands. This topography is a common bifurcation when breaking flow symmetries. Material in each of the period-5 islands and the centermost island is confined by KAM surfaces. Transport across these surfaces in a Newtonian fluid is diffusion-limited and in a suspension occurs either through diffusion or migration. This underlying structure is apparent in $\lambda=1\times10^{-4}$, where each island is close to $\phi_{\text{ave}}=0.2$, and is surrounded by higher concentration regions. These islands still exist at $\lambda=2.5\times10^{-3}$, but the positions of the period-5 regular regions shifted slightly counterclockwise. A center ring of high concentration is more pronounced and the boundaries share profile characteristics with $T=3/4$ and $\lambda=2.5\times10^{-3}$. At $T=3/2$ and $\lambda=1\times10^{-2}$ the concentration profile is most similar to $T=3/4$ and $\lambda=1\times10^{-2}$—the exception to this similarity is that the center region is slightly higher in concentration and shifted to the right.

At $T=5/2$ for $S_1$ [Fig. 3(i),(l)] the flow is more chaotic, with the topology again dominated by regular regions, islands to the right of the center are much smaller than those seen at lower $T$, and two sets of larger period-3 islands. One set of period-3 islands has one left island and two additional islands located near the top and bottom chaotic regions following close to the boundaries. The other set of period-3 islands, located in a chaotic region bounded by a KAM surface and completely surrounded by the chaotic region near the boundaries, has two islands near the left and one island near the right boundaries. In this concentration map shear migration does not follow the underlying topology as closely as in flows with shorter $T$. The concentration gradients are increasingly striated and the concentration is lower within the internal chaotic region; two lobes of lower concentration extend from the center to the left corners suggesting the bounding KAM surface inhibits migration into this region. At $\lambda=2.5\times10^{-3}$ the concentration profile is the inverse of that at $\lambda=1\times10^{-4}$. The concentration in the center region is higher, two bands of higher concentration extend from the center toward the left corners, and one band extends toward the lower right corner. Near the boundaries the concentration profile is similar to $T=3/4$ and $3/2$ at $\lambda=2.5\times10^{-3}$. The segregation patterns at the corners feed particles into the interior chaotic region. Stronger concentration gradients exist at $\lambda=1\times10^{-2}$ and the center region of high concentration is more pronounced than at $T=3/2$ and shifted further to the right.

At $T=6$ [Figs. 3(m)–3(p)] in the Poincaré map chaotic advection dominates everywhere except in a small group of islands located to the left. Many bifurcations occur in the range $5/2<T<6$ and are too numerous to make practical an elaboration of the entire evolution. The concentration profile at $\lambda=1\times10^{-4}$ has many striations including a larger band of higher concentration circulating upward from the lower left corner. With increasing $\lambda$ the suspension segregates more. At $\lambda=1\times10^{-2}$ the concentration profile rotated $180^\circ$ is similar to the steady profile shown in Fig. 2(d) because this flow spends a significant amount of time in each half cycle and is the limiting case as $T \to \infty$.

A similar analysis is performed for $S_2$ in Fig. 4. This flow generates the exact same instantaneous shear rate distribution as $S_1$, except that boundary conditions have horizontal reflectional symmetry across the $x$ midplane. This broken symmetry results in $180^\circ$ rotationally symmetric topology. In Fig. 4(a) the Poincaré map for $T=3/4$ shows two regular regions separated by an invariant surface extending from the right to left boundary. Circulation is clockwise in the upper regular region and counterclockwise in the lower regular region. The regular regions are smaller at $T=3/2$ [Fig. 4(e)] and surrounded by higher period island chains and a chaotic region extending to the boundaries. For $T=5/2$ [Fig. 4(i)], these regular regions are smaller and each has surrounding large period-4 islands. At $T=6$ [Fig. 4(m)] the original regular regions are no longer discernible with two distinct sets of period-2 islands taking their place.

Concentration profiles for $S_2$ is significantly different than those for $S_1$ because of the transport from the right to the left boundary through the center of the cavity. Figure $4(b)$, $\lambda=1\times10^{-4}$, shows lower concentration at the right and left boundaries as well as across the center in a horizontal jet. On either side of the jet $\phi_{\text{ave}}>0.2$ bounding the two regular regions with $\phi=0.2$. The topology of the Poincaré map dominates the shape of the segregation profile at low $T$. For $T=3/2$ and $5/2$ at $\lambda=1\times10^{-4}$ [Figs. 4(f) and 4(j), respectively] the center jet of lower concentration buckles but does not enter the area corresponding to the Poincaré map regular regions. At $T=6$ [Fig. 4(n)], the jet is located far from the center near the upper wall and between the two upper islands. This buckling breaks the rotational symmetry of the underlying topology.

For stronger migration at $\lambda=2.5\times10^{-3}$ [column 3, Figs. 4(c), 4(g), 4(k), and 4(o)] the concentration profile has stronger concentration gradients with similar topology to $\lambda=1\times10^{-4}$. The higher concentration regions form near the corners as in $S_1$, $\lambda=2.5\times10^{-3}$ and $1\times10^{-2}$. For very strong migration, $\lambda=1\times10^{-2}$, the jet of lower concentration is less clear [right column, Figs. 4(d), 4(h), 4(l), and 4(p)]. These concentration profiles, if reflected across the $x$ midplane, have similar features to Fig. 2(d) and thus demonstrate that unlike in $S_1$ the underlying flow topology has little effect on demixing regardless of $T$.

The effect of $T$ on demixing in each concentration profile can be quantified by calculating the intensity of segregation. Intensity of segregation, used as a tool to compare these results across the various flow profiles, is defined as

\[
I = \frac{\sigma^2}{\phi_{\text{ave}}(\phi_m - \phi_{\text{ave}})}
\]

based on the definition given by Danckwerts,\textsuperscript{37} where $\sigma$ is the standard deviation of $\phi$. $I=0$ indicates perfect mixing and $I=1$ occurs for perfect segregation. Figure 5 shows $S_1$ and $S_2$ at $\lambda=1\times10^{-2}$ and $\phi_{\text{ave}}=0.2$. $I$ is normalized by $I_0$ the inten-
steady lid-driven cavity flow. ence to reach a steady profile similar to that found in the high concentration island seen in Figs. 3 and 3(f). This island is not formed in S2 and thus chaotic advection only reduces segregation resulting from shear migration. Both S1 and S2 have asymptotes at $I \approx 0.0225$ as $T \to \infty$ because each half of the period is long enough to reach a steady profile similar to that found in steady lid-driven cavity flow.

Finally, we investigate the effect of $\phi_{\text{ave}}$ on mixing and shear migration in this flow. Figure 6 shows $0.1 \leq \phi_{\text{ave}} \approx 0.4$ for $S_1$, $T=3/2$. Shear migration-driven segregation is nonexistent at $\phi_{\text{ave}}=0$ and $\phi_{\text{ave}}=\phi_m$, and is strongest at an intermediate $\phi_{\text{ave}}$. For $\phi_{\text{ave}} \approx 0.3$ [Figs. 5(a)–5(c)] the underlying topology shown in Fig. 3(e) influences the segregation pattern and $\phi > \phi_{\text{ave}}$ surrounds the period-5 and central islands. For $\phi_{\text{ave}} \approx 0.3$ migration to the center of the cavity effectively washes out the period-5 islands. As $\phi_{\text{ave}} \to \phi_m$, the computation becomes unstable due to lack of resolution; in this limit the underlying flow varies significantly due to the increasingly non-Newtonian behavior of the suspension.

IV. DISCUSSION

The interplay between 2D time-periodic chaotic advection in a lid-driven cavity and shear migration of a suspension is highly complex. Although chaotic advection of a passive scalar in a Newtonian flow is complex in itself, generally mixing is enhanced when $T$ is large. Though $S_1$ and $S_2$ only differ in the translation direction of the bottom wall, this difference has a large effect on resulting flow topology. Despite this large difference in topology, the steady concentration profile in all convective-diffusive Newtonian flows is uniformity without concentration gradients. In Newtonian flows, the flow topology structure only defines the rate of dispersion. In contrast, shear gradients incur demixing in suspensions. For weak shear migration, $\lambda \approx 0$, the underlying flow topology defines the structure of the concentration gradients. At low $T$, $S_1$ produces a single recirculating flow with structure topologically similar to that of a steady lid-driven cavity. Because shear migration is strongest near the walls $S_1$ effectively has twice the surface area acting to shear fluid and $I$ is roughly twice that of the steady case. In contrast $S_2$ produces two recirculation regions at low $T$ with fluid convecting across the middle from the right to left boundary. Concentration gradients produced near the wall are advected into the center and as a result $I$ is roughly half that of the steady flow. Consequently, the same energy input into the system the flow can either enhance or reduce segregation. Similarly, at higher $T$, the underlying topology has a much larger area covered by chaotic trajectories. However, the interplay with migration is such that for $T \to \infty$, $S_1$, $S_2$, and steady flow converge on the same intensity of segregation. Both of these results contradict the perception that increasing the degree of chaos in the flow will enhance overall mixing. As computational resources develop calculating these concentration profiles via a normal stress balance will include the effects of curvature-driven migration. To develop a full set of heuristics for the influence of shear migration on mix-

![Graph](image1)

**FIG. 5.** (Color online) Intensity of segregation $I/I_0$ vs period $T$ for $S_1$ (diamonds) and $S_2$ (circles) at $\lambda = 1 \times 10^{-4}$ and $\phi_{\text{ave}}=0.2$. The dotted line indicates $I/I_0$ in the steady lid-driven cavity under the same conditions equivalent to $T \to \infty$.

![Concentration Profiles](image2)

**FIG. 6.** (Color online) Concentration profiles plotted at the end of each cycle of $T=3/2$, $\lambda = 1 \times 10^{-4}$, and $\phi_{\text{ave}}=0.1$ (a), 0.15 (b), 0.25 (c), 0.3 (d), 0.35 (e), and 0.45 (f). The scalebar of each profile has been adjusted to enhance the resolution of the segregation structure. There is no segregation in the limits $\phi_{\text{ave}} \to 0$ and $\phi_{\text{ave}} \to \phi_m$. 

Author complimentary copy. Redistribution subject to AIP license or copyright, see http://phf.aip.org/phf/copyright.jsp
ing and segregation in chaotic flows, a broader class of flows needs to be explored, especially 3D flows and open flows. Here, chaotic advection does not enhance dispersion because the regular regions do not correspond to high shear regions. Thus, designing flows to intentionally correlate the invariant structures with local shear to enhance mixing or separation may be possible. This analysis could develop mixing and separations on microscale platforms that depend only on suspension rheology eliminating the need to incorporate complicated MEMS to achieve desired concentration profiles. This is of particular interest because, in small scale flows, $\lambda$ is often relatively large and the time scales associated with chaotic mixing are size invariant when $\text{Pe} \gg 1$. Therefore, strategies employed at small scales that efficiently mix Newtonian fluids are often improper for suspension mixing.\textsuperscript{15,16} Similarly, efficiency of particle separations at small scales are enhanced for increased $\lambda$.

**ACKNOWLEDGMENTS**

We acknowledge extensive discussions with J. F. Morris. This work was supported by the North American Mixing Forum.


\textsuperscript{19}A. A. Abatan, J. J. McCarthy, and W. L. Vargas, “Particle migration in the rotating flow between co-axial disks,” AIChe J. \textbf{52}, 2039 (2006).


\textsuperscript{35}A. J. Lichtenberg and M. A. Lieberman, Regular and Stochastic Motion (Springer, New York, 1983).

\textsuperscript{36}H. C. Hilborn, Chaos in Nonlinear Systems (Oxford University Press, New York, 1994).